INTRODUCTION

In [6], reasons are given for enriching the conventional language of modal logic with an operator ‘A’ (read ‘actually’) whose function is to effect (loosely speaking) a reference to a single world (within a model) designated as the actual world. The need for such an operator is illustrated by the unrepresentability, without it, of sentences of the form ‘It is possible for everything which is in fact φ to be ψ’, which, with its aid can be rendered:

\[ \Diamond (\forall x) (A \phi x \rightarrow \psi x). \]

For a full discussion, together with references to related work, see [6] (henceforth LA), Section 1, and [10], Sections 1 and 3. The obvious semantical account of the logic of ‘actually’, when one has in mind S5 as the underlying modal logic, utilises models of the form \( (W, w^*, V) \) where \( w^* \in W \) functions as the actual world of the model, and \( V \), which assigns truth-values to propositional variables paired with elements of \( W \), is extended to the general truth-relation \( \models \) by the usual clauses for the truth-functional connectives and (with no relational restriction) for ‘\( \Box \)’. The clause for ‘A’ reads, not surprisingly, as follows: for any model \( \mathcal{W} = (W, w^*, V) \) and any \( x \in W \), \( \mathcal{W} \models_x A \alpha \) iff \( \mathcal{W}_{w^*} \models \alpha \). In calling a formula S5A-valid, we mean that it is false at no world in any model. This notion of validity (general validity) should be distinguished from another: a formula is real world valid if for no model is it false at the actual world of the model. If \( \alpha \) is real world valid then, of course \( A \alpha \) is generally valid. One reason for using the notion of general validity is that in an axiom system which yields as theorems precisely the generally valid formulae the substitutivity of provable equivalents holds.1

While the interest of this enriched language arises largely through the interaction of ‘\( \Box \)’, ‘A’ and the quantifiers, the technical novelties all emerge at the...
purely propositional level, so that it is on this that we concentrate here. In LA, it is shown that the class of S5A-valid formulae coincides with the class of theorems of the system S5A, axiomatized by adding to any set of axiom-schemata sufficient for propositional S5, together with Modus Ponens and Necessitation (henceforth 'Nec.') as rules of proof, the following axioms:

(A1) \( A(\alpha \rightarrow \beta) \rightarrow (A\alpha \rightarrow A\beta) \)
(A2) \( A\alpha \rightarrow \sim A \sim \alpha \)
(A3) \( \Box \alpha \rightarrow A\alpha \)
(A4) \( A\alpha \rightarrow \Box A\alpha \)

Although (A4), like the other axioms, is easily seen to be valid on the semantics offered, it is apt to arouse suspicion on the grounds that since 'actually \( \alpha \)' is true (in any world) because \( \alpha \) is true in the actual world, 'actually \( \alpha \)' need not have been true because another world might have been actual. To do justice to this intuition, in LA it is suggested that one consider an alternative to '\( \Box \)' for the representation of necessity, this alternative formalization being the modal prefix ' \( \mathcal{F} A \)' where ' \( \mathcal{F} \)' is the actuality operator whose semantics have just been explained and ' \( \mathcal{F} \)' (for 'fixedly') is a new operator whose semantical clause is stated in terms of the relation \( \approx \) of variance between models differing at most over which world is designated as the actual world. The clause runs:

\[ \mathcal{W} \models_{\mathcal{F}} \alpha \text{ if for any } \mathcal{W}' \approx \mathcal{W}, \mathcal{W}' \models_{\mathcal{F}} \alpha. \]

In fact, once we have the relation \( \approx \) available, we can define four necessity-like operators:

\[ \mathcal{W} \models_{\Box} \alpha \text{ if for any } y \in W, \mathcal{W} \models_{ y } \alpha \]
\[ \mathcal{W} \models_{\mathcal{F}} \Box \alpha \text{ if for any } \mathcal{W}' \approx \mathcal{W}, \mathcal{W}' \models_{\mathcal{F}} \alpha \]
\[ \mathcal{W} \models_{\mathcal{F}} \alpha \text{ if for any } \mathcal{W}', \mathcal{W}' \approx \mathcal{W}, \text{ if } \mathcal{W}' = (W, y, V) \text{ then } \mathcal{W}' \models_{\mathcal{F}} \alpha \]

In the case of ' \( \Box \)' we consider only changes in the world at which the formula is evaluated. In the case of ' \( \mathcal{F} \)' we consider only changes in which world is designated as actual. In the case of ' \( \mathcal{F} \)' we consider changes of both kinds but require that the formula be evaluated at the world designated as actual. In the case of ' \( \mathcal{F} \)' we consider independent changes of both kinds. ' \( \Box \)' is then definable as ' \( \Box \)' , ' \( \mathcal{F} \)' as ' \( \mathcal{F} A \)' , ' \( \mathcal{F} \)' as ' \( \mathcal{F} \) \', and ' \( \mathcal{F} \)' as ' \( \mathcal{F} \Box \)'.
Thus ‘$\forall \alpha$’ says: whichever world had been actual, $\alpha$ would have been true at that world considered as actual. The original modal operator ‘$\Box$’ remains important, for it was precisely in order to interact with ‘$\Box$’ that ‘$A$’ was introduced. The difference between ‘$\forall A$’ and ‘$\Box$’ turns out to be a formal rendering of a distinction invoked by Gareth Evans between deep and superficial necessity, which Evans has used to cast light on the problem of the contingent a priori, in [8] (henceforth RC). In LA, the logic of ‘$\forall$’ was left as an open problem; we sort it out in Section 1 of the present paper and in [11] where a completeness proof in the style of [15] is provided. In the rest of the present paper we present applications of the machinery developed to the topics of the contingent a priori and the necessary a posteriori, as well as going into the question of what a truth theory for a language with ‘$\forall$’, alongside ‘$\Box$’ and ‘$A$’ looks like. This last matter deserves attention because the availability of a homophonic truth theory is a necessary condition for the truth of the claim that the modal notions in question are intelligible in their own terms, without recourse to the apparatus of possible worlds in terms of which the model theory is presented. In addition, a theory of truth for the full language with ‘$\Box$’, ‘$A$’, and ‘$\forall$’ illuminates the relation between the notion of truth with respect to a world or possible situation, a notion which is, as Evans says, ‘internal to the semantic theory’, and the notion of truth in a possible situation as what sincere assertion in that situation aims at (what it would be true to say, were such and such the case). This is another distinction emphasized by Evans in RC. The present paper is thus both a sequel to LA and a supplement to RC.

1. LOGICAL PRELIMINARIES

If we keep the definition of validity given in the previous section, but take it to apply now to formulae of the language with ‘$\forall$’, as well as ‘$\Box$’ and ‘$A$’, we get what we may call S5A.$\forall$-validity. The following rule of proof and axioms, when added to the basis given above for S5A, yield a system we call S5A.$\forall$ which has as its theorems all and only the S5A.$\forall$-valid formulae.

\[
\begin{align*}
(\forall 1) & : \forall (\alpha \to \beta) \to (\forall \alpha \to \forall \beta) \\
(\forall 2) & : \forall \alpha \to \alpha \\
(\forall 3) & : \forall \alpha \to \forall \forall \alpha \\
(\forall 4) & : \alpha \to \forall \sim \forall \sim \alpha \\
(\forall 5) & : \alpha \to \forall \alpha \text{ for any } A\text{-free formula } \alpha.
\end{align*}
\]
(F 6)  \( \square \alpha \leftrightarrow \Box \alpha \) for any \( A \)-free formula \( \alpha \).

Rule: from \( \neg \alpha \) to \( \neg \square \alpha \) (Call this rule 'Fix'.)

Observe that the rule Fix., together with (F1)–(F4) give \( \square \) an S5-style logic. (One expects this, since variance between models is an equivalence relation.) In the completeness proof of [11] the consequences of these first four axioms that we exploit are

\[
\begin{align*}
(T1) & \quad \square (\alpha \land \beta) \leftrightarrow (\square \alpha \land \square \beta) \\
(T2) & \quad \square (\alpha \lor \beta) \leftrightarrow (\square \alpha \lor \square \beta).
\end{align*}
\]

In verifying the validity of (F5) and (F6) we make use of the fact that if \( \alpha \) contains no occurrence of 'A' and \( \mathcal{W} \) and \( \mathcal{W}' \) are variants, then \( \mathcal{W}' \models_x \alpha \) iff \( \mathcal{W} \models_x \alpha \). In the case of (F6) we also need to use the fact that any world in a model is the designated world in some variant of that model. (F6) ties together \( \square \) and \( \Box \): if not every world could be considered as the designated world the axiom would fail in its left-to-right direction, while if the clause for \( \Box \) did not involve quantification over all the worlds in the model, it would fail from right to left. Enough has been said to demonstrate the soundness of S5A\( \mathcal{F} \) with respect to the semantics given. Completeness is proved in [11] via the following Elimination Theorem for \( \square \): For any formula \( \alpha \) there exists a formula \( \alpha' \) not containing \( \square \) such that S5A\( \mathcal{F} \mod \alpha \models \alpha' \).

Before pursuing the logic of S5A\( \mathcal{F} \) further, we pause to note that a slightly different way of dressing up the semantical ideas so far presented is available: that is, we could use the notation and terminology of what has come to be called two-dimensional modal logic.4 Within such a framework, one uses a doubly-indexed truth relation, letting the upper index function as the actual world, instead of having this given as part of the identity of the model in question; writing, instead of the clause

\[
\langle W, w^*, V \rangle \models_x A \alpha \iff \langle W, w^*, V \rangle \models_{w^*} \alpha
\]

the following:

\[
\langle W, V \rangle \models^V_x A \alpha \iff \langle W, V \rangle \models^V_y \alpha.
\]

(Read \( \models^V_x \alpha \), for present purposes, as: \( \alpha \) is true at \( x \) from the perspective of \( y \) as the actual world.) This is the only place at which the upper index gets into the act in S5A, though in S5A\( \mathcal{F} \) the new operator \( \mathcal{F} \) is able to quantify into upper index position just as \( \Box \) quantifies into the position of
the lower index. We shall continue to use the approach outlined in Section 0, however, with its designated worlds and variance relation, except for some informal remarks in Section 3.5

Since we shall, in Section 3, be looking into the question of how to construct a truth-theory for the language of S5A, we must establish here what conditions are required for the substitutivity of equivalents. It turns out, not surprisingly perhaps, that the prefix 'A' justifies substitutions. By this we mean not just that all instances of (SE):

\[(SE) \quad A(\alpha \leftrightarrow \beta) \rightarrow (\gamma \leftrightarrow \gamma')\]

are provable in S5A, where \(\gamma\) is any formula containing \(\alpha\) as a subformula and \(\gamma'\) differs from \(\gamma\) in having one or more occurrence of \(\alpha\) replaced by \(\beta\): this follows merely from the fact that each of 'A', 'A' has a modal logic at least as strong as the weak system K. Rather, because we need in the truth-theory to make substitutions on the basis of equivalences themselves resulting from such substitutions, we require the stronger condition that all instances of \((SE^+)\) should be provable:

\[(SE^+) \quad A(\alpha \leftrightarrow \beta) \rightarrow A(\gamma \leftrightarrow \gamma')\]

(it would be a mistake to think that we simply use the relevant instance of (SE), detach the consequent and apply Nec. and Fix. to the result: these are rules of proof in the background logic to the truth theory, not rules of inference for use in any (non-logical) theory.) For either 'A' or 'A' alone, the step from something corresponding to (SE) to something corresponding to \((SE^+)\) would be an immediate consequence of the fact that each operator has at least as strong a logic as the system S4, but this fact does not suffice for the provability of (T3):

\[(T3) \quad A\alpha \rightarrow A A A\alpha\]

which would get us from (SE) to \((SE^+)\), as one sees by considering the falsifiability of 'O\(_1\)O\(_2\)p \rightarrow O\(_1\)O\(_2\)O\(_2\)p' in a Kripke model \((W, R_1, R_2, V)\) in which \(R_i\) interprets the operator \(O_i\) and both \(R_i\) are reflexive and transitive.6 However, (T3) is a theorem(schama) of S5A, as it follows from the S4-necessity of 'A' and 'A', together with a substitution licensed by (T4):7

\[(T4) \quad A\alpha \leftrightarrow \Box A\alpha\]

We close this discussion of substitutivity by pointing out that while either
of the equivalent prefixes ‘\(\mathcal{F} \Box\)’ and ‘\(\Box \mathcal{F}\)’ will justify substitutions (in the sense of \((SE^+)\)), it is definitely the concatenation of the two operators and not their conjunction that is required. The reader will have no difficulty in constructing a two element model which falsifies, for example:

\[
(\mathcal{F} (Ap \lor q) \land \Box (Ap \lor q)) \rightarrow \mathcal{F} \Box (Ap \lor q)
\]

This amounts to denying substitutivity even in the weak form (i.e., the form corresponding to \((SE)\)) because, taking ‘\(Ap \lor q\)’ as \(\alpha\) and abbreviating any valid formula to ‘\(T\)’, it gives a counterexample to:

\[
(\mathcal{F} (\alpha \leftrightarrow T) \land \Box (\alpha \leftrightarrow T)) \rightarrow (\mathcal{F} \Box \alpha \leftrightarrow \mathcal{F} \Box T).
\]

Our final remarks on the logic of S5A\(\mathcal{F}\) pertain to the claim made in \(LA\) and endorsed here that ‘\(\mathcal{F} \alpha\)’ embodies a notion of necessity. This suggests the question: does ‘\(\mathcal{F} \alpha\)’ considered as a single modal operator itself have an S5 logic? The answer to this question is that the S4 and S5 axioms for ‘\(\mathcal{F} \alpha\)’

\[
\begin{align*}
\mathcal{F} \alpha & \rightarrow \mathcal{F} \mathcal{F} \alpha, \\
\sim \mathcal{F} \alpha & \rightarrow \mathcal{F} \mathcal{F} \alpha
\end{align*}
\]

are both provable, but the T-axiom for ‘\(\mathcal{F} \alpha\)’

\[
\mathcal{F} \alpha \rightarrow \alpha
\]

is not provable. Indeed it is easy to see that the T-axiom is not valid: take, for example, \(Ap \leftrightarrow p\) for \(\alpha\). Does this do serious damage to the claim that ‘\(\mathcal{F} \alpha\)’ expresses a notion of necessity? Not really. For any model \(\mathcal{W} = (W, w^*, V)\),

\[
\mathcal{W} \models_{w^*} \mathcal{F} \alpha \rightarrow \alpha,
\]

from which it follows that

\[
A (\mathcal{F} \alpha \rightarrow \alpha)
\]

and its equivalent

\[
\mathcal{F} \alpha \rightarrow A \alpha
\]

are valid, and these arguably answer well to the principle \textit{ab necesse ad esse}. We shall see later the philosophical uses to which this notion of necessity can be put.
2. THE CONTINGENT ‘A PRIORI’

In *RC*, Evans argues that “there is nothing particularly perplexing about the existence of a statement which is both knowable *a priori* and superficially contingent” (p. 161), where a sentence $\sigma$ is *superficially contingent* just in case $\Box (\neg \sigma)$ and $\Box (\neg \neg \sigma)$ are both false. Some standard examples of statements which are knowable *a priori* though contingent are expressed using what Evans calls *descriptive names*. Consider, for example, ‘Julius’, a syntactic name whose reference is fixed by the description ‘the inventor of the zip’. ‘Julius’ is to have two features which initially might seem to be incompatible:

(i) ‘Julius’ is sufficiently similar to an ordinary proper name to be regarded as a referring expression, even though definite descriptions are not regarded as referring expressions.

(ii) One can understand sentences containing ‘Julius’ without knowing of any object that it is being said to be thus and so.

It certainly is possible to use ‘refers’ in such a way that (i) and (ii) are incompatible, but to insist on such a use would be pointless at this stage; rather we should seek to understand the semantic function of descriptive names.

The reason why (i) and (ii) might seem to be incompatible is that on one rather plausible view proper names differ from definite descriptions in two respects; and (i) requires descriptive names to be like proper names in one respect while (ii) requires descriptive names to be like definite descriptions in the other. For, on that plausible view, a proper name is assigned an object which is salient in two distinguishable ways. First, the object is truth conditionally salient (tc-salient); that is, the truth with respect to counterfactual situations of sentences containing the name, and so the truth of modal sentences containing the name, depends upon how things are with that object. Second, the object is epistemologically salient (e-salient); that is, in order to understand sentences containing the name one must have *de re* propositional attitudes concerning that object, and in the case of a material object must stand in some appropriate causal relation to that object. In contrast, the object assigned to a definite description such as ‘the inventor of the zip’ is salient in neither of these ways. What is puzzling about descriptive names is that if a descriptive name is non-empty then the object which is assigned to it is tc-salient but not e-salient. Thus suppose that, in fact, Tom uniquely invented the zip, and that we are out to describe the way in which
'Julius' functions in atomic sentences and their □-modalizations. Then Tom is tc-salient in that the truth of □F(Julius) or of □□F(Julius) turns on how things are with Tom. In this respect, 'Julius' is like a proper name of Tom and unlike a definite description -- or rather, it is unlike the A-free description 'the inventor of the zip'. But Tom is not e-salient since to understand those sentences one does not need to have any de re propositional attitudes concerning Tom, and in particular does not need to stand in any interesting causal relation to that man. In this respect 'Julius' is unlike a proper name and like a definite description.

The sentence

(S) If anyone uniquely invented the zip, Julius invented the zip.

is true, but the □-modalization of (S) is false. So (S) is a (superficially) contingent truth. On the other hand, to understand (S) is merely to understand that it states that if anyone uniquely invented the zip, then whoever invented the zip did. So the statement expressed by (S) is knowable a priori. One way of putting the puzzle, then, is this. An outright assertion using (S) intuitively has the same content as an assertion using:

(S') If anyone uniquely invented the zip, the inventor of the zip invented the zip.

Yet (S) and (S') embed differently in □-contexts: (□S) is false while (□S') is true.

Evans' solution to this puzzle is best seen if one employs possible worlds terminology with a fixed actual world w* (the actual actual world) playing the role of the actual world throughout. Associated with (S) is the property:

\[ \lambda w (\text{if anyone uniquely invented the zip in } w \text{ then whoever invented the zip in } w* \text{ invented the zip in } w) \]

The actual world w* has this property, but some other worlds lack it. Associated with (S') is the following property, which no world lacks:

\[ \lambda w (\text{if anyone uniquely invented the zip in } w \text{ then whoever invented the zip in } w \text{ invented the zip in } w) \]

This is why (S) and (S') embed differently in □-contexts. On the other hand, an outright assertion of either (S) or (S') in the actual world has the force of
an assertion that \( w^* \) has the associated property, and \( \lambda \)-conversion reveals that these two assertive contents are precisely the same.

While a sentence such as (S) is superficially contingent, Evans distinguishes a notion of *deep contingency* which does not apply to (S). Of this notion he says, "If a deeply contingent statement is true, there will exist some state of affairs of which we can say both that had it not existed the statement would not have been true, and that it might not have existed" (p. 185). Evans' solution to the puzzle strikes us as both elegant and convincing. In particular, it is doubly preferable to any solution based on an attempt to show that there is something wrong with the notion of a descriptive name, by showing that if 'Julius' is anything other than an abbreviation for 'the inventor of the zip', then it is a proper name mastery of which requires knowledge *a posteriori* concerning the man Tom that he is its referent (in which case mere knowledge of the reference-fixing description does not suffice for knowledge of anything which can be expressed by a *use* of the name 'Julius'). For first, as Evans argues in Section I (of *RC*), the notion of a descriptive name certainly is coherent — whatever one's feelings about the extent to which such a category is manifested in natural languages, and second, as he points out in Section IV, essentially similar examples of the contingent *a priori* can be provided without appeal to expressions of that supposedly dubious kind.

We should like to take up this second point, urging that in general the notion of the contingent *a priori* is well approached via the rather modest logical apparatus set up in Sections 0 and 1 of the present paper. For within propositional S5\( \mathcal{A} \mathcal{F} \) one can provide uncontroversial examples of the contingent *a priori*, and mark out formally the distinction between superficial and deep contingency. One can know *a priori* that grass is actually green iff grass is green, although this is a superficially contingent truth: if \( \square \sigma \) and \( \sigma \) are true then so also is \( \neg \square (A \sigma \leftrightarrow \sigma) \). On the other hand, \( \square \mathcal{F}A (A \sigma \leftrightarrow \sigma) \) is true for any sentence \( \sigma \), for in general \( \square \mathcal{F}A \tau \) is true just in case whatever world is actual, \( \tau \) is true with respect to that world: that is, just in case every world has the property which \( \tau \) requires of a world *considered as actual*. What \( A \sigma \leftrightarrow \sigma \) requires of a world considered as actual is nothing at all, although if the actual world is already fixed then what is required of a world \( w \) is that \( w \) agree with the actual world in the truth-value assigned to \( \sigma \). So our suggestion is that just as a true sentence \( \tau \) is superficially contingent iff \( \square \tau \) is false, so a true sentence \( \tau \) is deeply contingent iff \( \mathcal{F}A \tau \) is false,
and we shall argue in the next section that this is connected in the right way with \textit{what makes a sentence true}. 

Perhaps the ‘deep’/’superficial’ terminology invites the thought that although some superficially contingent truths are not deeply contingent, still every deeply contingent truth is also superficially contingent. If so, then the invitation should be declined: it is, for example, deeply but not superficially contingent that grass is actually green. (Recall that \( \Box A \sigma \) is equivalent to \( \Box A \sigma \), while \( \Box \mathcal{F} (A \sigma) \) is equivalent to \( \Box \mathcal{F} A \sigma \) and, if \( \sigma \) is \( A \)-free, to \( \Box \sigma \). Here we have a fund of simple examples of the necessary \textit{a posteriori}: one can know only \textit{a posteriori} that grass is actually green.)

There is an obvious third notion of contingency, which includes both the others. Let us say that a true sentence \( \sigma \) is \textit{bi-contingent} iff \( \Box \neg \Box \mathcal{F} \sigma \) is true, that is, iff there are worlds \( w_1 \) and \( w_2 \) such that with \( w_1 \) considered as actual, \( \sigma \) is false with respect to \( w_2 \). The inclusion is proper, for it is neither deeply nor superficially contingent that if grass is actually orange then grass is orange, yet that is clearly a \textit{bi-contingent} truth.

It would be natural to ask whether all and only deeply necessary truths are knowable \textit{a priori}. The question is a difficult one. First, it is not obviously correct to say that all deeply necessary truths are knowable \textit{a priori}, for example because true identity statements using proper names are both superficially and deeply necessary (since such names are not even implicitly ‘\( A \)’-involving) whereas it is not clear that such statements are \textit{a priori} knowable: one’s view here would depend on one’s views about the sense of proper names. Further, as we shall see below, the axioms of a truth-theory for a language are deeply necessary (or must be taken to be so for the theory to be adequate when the language contains ‘\( \Box \)’, ‘\( \mathcal{F} \)’, and ‘\( A \)’) whereas many people would feel that such axioms are obviously not \textit{a priori} knowable; however, here again, the answer is sensitive to some delicate issues of language individuation and the semantics of quotation. As to the converse, we can only report here that we have not yet noticed any examples of truths expressed in terms of ‘\( \Box \)’, ‘\( \mathcal{F} \)’, and ‘\( A \)’ which are \textit{a priori} knowable and not also deeply necessary. For the record, we should remark that the process which yielded 2-dimensional modal logic from the more familiar 1-dimensional kind can be iterated: truth can be triply relativized to a \textit{real} actual world \( w_1 \), a ‘floating’ actual world \( w_2 \), and a floating reference world \( w_3 \). If we introduce an operator ‘\( R \)’ such that \( R \alpha \sigma \) is true with respect to such a triple \( (w_1, w_2, w_3) \) iff \( \alpha \) is true with respect to \( (w_1, w_1, w_3) \), then \( R A \sigma \leftrightarrow A \sigma \) will in general
be deeply contingent on the present 'FA' definition since that definition is
tailored to fit the 2-dimensional case, as also will be \([RA \sigma \leftrightarrow \sigma]\). Yet such
sentences express a priori truths. Naturally, a new notion of deep contingency
can be defined for the 3-dimensional case, which will in turn allow a priori
contingent truths expressible with 4-dimensional operators, and so on. We
shall make no attempt to deal with this matter in full generality here.

Although there are certain difficulties in making the transition from
propositional to quantificational S5A\(\mathcal{A}\) -- difficulties arising out of the need
to allow for world-to-world variation of domain in the model theory\(^{18}\) --
these difficulties have nothing to do with our present concerns, and it is
intuitively clear that cases of the contingent a priori will arise when we
consider definite descriptions including ‘actually’. In particular, consider:

\[(S'')\quad \text{If anyone uniquely invented the zip, the actual inventor of the}
\text{zip invented the zip.}\]

What \((S'')\) states can be known a priori and yet \((S'')\) is superficially con-
tingent, since the property of worlds associated with \((S'')\) is just the property
associated with \((S)\). On the other hand, the \(\mathcal{A}\)-modalization of \((S'')\) is
true: \((S'')\) is deeply necessary. This shows that the initially puzzling features
of \((S)\) can be reproduced without using descriptive names like ‘Julius’, since
\((S'')\) involves a straightforwardly quantifier-like definite description.\(^{19}\) These
considerations suggest indeed that a descriptive name with its reference fixed
by ‘the G’ is nothing other than a conventional abbreviation of (or at least, an
expression whose sense is that of) ‘the actual G’. Such a suggestion has the
advantage of promising a uniform explanation of the contingent a priori: it
is occasioned by the presence of ‘actually’ (or some equivalent). Whether the
suggestion ultimately proves to be tenable would depend on the resolution
of such questions as: could a language containing unstructured expressions
functioning as descriptive names fail to contain anything corresponding to
‘actually’? In the meantime, consideration of the suggestion aids perception
of the vast gap between an argument against the coherence of descriptive
names and a solution to the problem of the contingent a priori.

Finally, we return to descriptive names themselves and ask whether they
are to be considered referring expressions in the strongest sense of the phrase,
a sense which, following Evans, we may take to be captured by the condition
that the semantic role of such an expression can be stated using a notion of
reference which is not world-relative. Evans himself returns an affirmative
answer to this question on the basis of the behaviour of descriptive names in \( \Box \)-contexts (in modal and counterfactual contexts, as usually conceived). This is a natural response on the basis of such \( \Box \)-context examples, since the descriptive name's referent may be thought of as determined once and for all as the actual world's satisfier of the description, regardless of the applicability of that description to that object in other worlds. But a negative answer to the question is suggested once we consider the richer variety of contexts available within a language containing ‘\( \mathcal{F} \)’ and ‘\( \mathcal{A} \)’ alongside ‘\( \Box \)’. There is, in the semantic theory for such a language, at least the possibility of doubly relativizing the notion of reference, so that reference would exhibit that double relativity (to an actual world and a ‘floating’ world) exhibited by truth. The reference relation for proper names requires no relativization, that for descriptions requires the full double relativization, while the reference relation for descriptive names requires relativization in just the actual world place. Evans does not make this threefold distinction because, in effect, he considers the actual world place in the reference relation and the truth predicate to be filled by a constant (‘\( w^* \)’) for the actual actual world. But this way of ruling out the type of relativization at issue is not obviously adequate to everything Evans himself says, for he wants to allow that if in world \( w \) Gough Whitlam invented the zip then although ‘Julius is Whitlam’ is not true with respect to \( w \) (not true\( _{w^*,w} \), as one might put it), nevertheless if \( w \) were actual then ‘Julius is Whitlam’ would be true — without linguistic change. (The ins and outs of the notion of truth will be our concern in the next section.)

There are at least two ways in which one might resist the negative answer to the question whether descriptive names are referring expressions. One way is to claim that the relativity of the reference of descriptive names is relativity to context, and to point out that such relativity does not disqualify an expression as a referring expression: ‘I’ and ‘you’ are so relative. On this view ‘\( \mathcal{F} \)’ is a context-shifting operator; what Kaplan calls a ‘monster’. This way of resisting could be encouraged by considering the formal parallel between modal logic and tense logic. In the temporal case the present moment — the moment of utterance — is a context, the relativity of the reference of ‘Now’ is context relativity, and the temporal operator corresponding to ‘\( \mathcal{F} \)’ is naturally considered a context-shifting operator. But, as Evans has pointed out in another place, the formal similarities must not be allowed to blind us to deep differences between tense and modality, and these differences...
seem to us to count against regarding the actual world as a context. Another way is to claim that descriptive names are not semantically similar to descriptions embedding the operator ‘A’ but similar to descriptions embedding a different operator ‘A*’ which takes us back to the actual world, even when ‘A*’ occurs within the scope of ‘F’. The difficulty which his strategy faces is that it must (a) explain why one could not introduce an operator ‘F*’ standing to ‘A*’ as ‘F’ stands to ‘A’, and (b) allow that had Whitlam invented the zip we could have said truly that Julius is Whitlam; and it must do both these things without simply collapsing into the first way. Perhaps this second way can succeed, and perhaps another way can be found; but our provisional view is that the negative answer is correct.

3. TRUTH

As indicated in Section 0, some interest attaches to the question of whether it is possible to give a homophonic truth theory for at least a propositional language containing ‘□’, and ‘A’, and ‘F’. No-one familiar with homophonic theories for ‘□’-languages and with the logic of ‘F’ set out in Section 1, will be surprised to learn that the □,□-modalizations of the disquotational truth condition specifying biconditionals for atomic sentences and truth-functional connectives, together with

\[(\theta 1) \quad F \Box [(\forall \sigma)(Tr(\Box \sigma) \iff \Box Tr(\sigma))]\]
\[(\theta 2) \quad F \Box [(\forall \sigma)(Tr(\neg A \sigma) \iff A Tr(\sigma))]\]

and

\[(\theta 3) \quad F \Box [(\forall \sigma)(Tr(\neg F \Box \sigma) \iff F Tr(\sigma))]\]

will suffice as axioms for the derivation of all instances of

\[F \Box [Tr(\neg Q) \iff Q]\]

and so all instances of

\[F A [Tr(\neg Q) \iff Q]\]
\[\Box [Tr(\neg Q) \iff Q]\]
\[A [Tr(\neg Q) \iff Q]\]

and the bare biconditionals.
when we take $S5A^T$ as our underlying logic.\(^{23}\) (Here and elsewhere we suppress the language parameter, while reminding the reader that it is the presence of this parameter which enables us to block what might otherwise seem to be an objection to any theory employing modalizations of truth condition specifying biconditionals.\(^{24}\) ) The model theory of Section 0, together with the notion of an intended model, yields a reading of these biconditionals in the heuristically useful terminology of possible worlds, and the reader might care to look over the axioms just given with such an interpretation in mind to check that they are indeed true.

Although $S5A^T$ as our underlying logic delivers from our proper axioms the desired biconditionals, it yields unfortunately some false theorems as well and must be restricted. For example, using ($\theta\ 2$) and ($\theta\ 3$), we have:

$$\mathcal{F}A(Tr(\overline{A \sigma \leftrightarrow \sigma})) \leftrightarrow \mathcal{F}A (A \sigma \leftrightarrow \sigma).$$

What follows ' $\mathcal{F}A$ ' on the left of this biconditional is $A$-free: ' $A$ ' is mentioned there but not used. So axiom ($\mathcal{F}6$) of Section 1 takes us to

$$\Box Tr(\overline{A \sigma \leftrightarrow \sigma}) \leftrightarrow \mathcal{F}A (A \sigma \leftrightarrow \sigma)$$

and so (with ($\theta\ 1$)) to

$$Tr(\overline{\Box (A \sigma \leftrightarrow \sigma)}) \leftrightarrow \mathcal{F}A (A \sigma \leftrightarrow \sigma)$$

which is, in general, false. Next, consider the biconditional

$$Tr(\overline{A \sigma}) \leftrightarrow A \sigma.$$ On the left of ' $\leftrightarrow$ ' we have an atomic sentence, so that axiom ($\mathcal{F}5$) would lead to

$$\mathcal{F} Tr(\overline{A \sigma}) \leftrightarrow A \sigma$$

and hence (using ($\theta\ 3$)) to

$$Tr(\overline{\mathcal{F}A \sigma}) \leftrightarrow A \sigma$$

which is again, in general, false. Thus, both of the schemata ($\mathcal{F}5$) and ($\mathcal{F}6$) must be subject to a further restriction if they are to be regarded as part of the meta-language's resources for deriving the truth conditions of object language sentences. This ought not to be too surprising, since, once the object
language contains descriptive names both schemata will, in any case, need to be restricted to take account of the fact that even sentences which do not explicitly use 'actually' may be such that their truth with respect to a world depends on how things are in the actual world. Meta-language sentences predicating truth of object language sentences are just another instance of the same phenomenon. In this case, as in others,

the truth values of semantic predicates in various possible worlds with respect to various objects must be allowed to fall where they may according to our best insights into the truth of object language sentences and constraints on the concepts of truth and satisfaction.\textsuperscript{35}

In fact, rather than restricting the two schemata in question, we suggest that the simplest course to adopt for present purposes would be to omit them altogether and elevate their consequence (T4) to axiomhood.\textsuperscript{26} It will be recalled from Section 1 that this principle (T4), which says that ‘\( \mathcal{F} \)’ and ‘\( \square \)’ commute, is important because it is needed for proving substitutivity in the form (SE\textsuperscript*) needed for the truth theory. Note also that it is a safe principle to add because, unlike (\( \mathcal{F}5 \)) and (\( \mathcal{F}6 \)) its validity in no way depends upon any particular way of drawing the \( \mathcal{A} \)-containing/\( \mathcal{A} \)-free distinction: indeed, semantically, (T4) just comes down to recording the fact that adjacent universal quantifiers commute.

It is instructive to ask how the output of the truth theory presented here is connected with the truth of utterances. (To avoid confusion, we shall capitalize the evaluative predicate of utterances: ‘TRUE’.) What one expects is that some modalization of

\[
\text{(T)} \quad \text{If } u \text{ is an utterance of sentence type } \sigma \text{ then } \text{TRUE} (u) \text{ iff } Tr(\sigma)
\]

together with the correspondingly modalized output of the theory will yield specifications of TRUTH conditions of utterances. What might look promising is to use the \( \mathcal{F} \square \)-modalizations, for if grass is green in the actual world \( w_1 \) and in \( w_2 \) grass is orange then surely an utterance in \( w_2 \) of ‘Grass is not green’ is TRUE and with respect to \( w_2 \) that sentence is indeed true. But this proposal is unsatisfactory because it fails to take into account the indexicality of ‘actually’: an utterance in \( w_2 \) of ‘Grass is actually green’ is FALSE. The problem here is that the \( \mathcal{F} \square \)-biconditionals specify under what conditions a sentence \( \sigma \) is true with respect to \( w_2 \), with \( w_1 \) as the actual world, and these conditions are not in general the conditions under which \( \sigma \)
is good for making a TRUE assertion in \( w_2 \). This is surely what Evans was pointing to when he remarked that if a certain state of affairs had obtained a sentence would have been true even though it is not true with respect to that situation (RC, p. 181). Similar difficulties obviously prevent the use of the \( \Box \)-modalizations, and since the \( A \)-biconditionals and the bare biconditionals do not involve any world relativity at all — and so could not contribute to the determination of the TRUTH conditions of utterances made in counterfactual situations — we are left with the \( F A \)-biconditionals and the \( F A \)-modalization of \((T)\). These deliver, for example, that for any world \( w \) as the actual world an utterance in that same world of ‘Grass is green’ is TRUE just in case that sentence is true with respect to \( w \), that is, just in case grass is green in \( w \), and an utterance in that same world of ‘Grass is actually green’ is TRUE just in case that sentence is true with respect to \( w \) for \( w \) as the actual world, that is, just in case grass is green in \( w \). In fact, since from the point of view of an utterer making an utterance in world \( w \) the actual world is just world \( w \), the \( F A \)-principles answer precisely to an interest in TRUE utterances. Reverting to a relativized truth predicate, and putting the matter contentiously, we can say that the truth which matters, the truth at which sincere asserters in \( w \) aim, is truth\(_{w,w}\); for a language whose only modal operator had the force of our ‘\( F A \)’ this would be the only truth which was semantically relevant, but to deal with ‘\( F \)’, ‘\( \Box \)’ and ‘\( A \)’ we need to consider — en route to truth\(_{w,w}\) conditions — truth\(_{w_1,w_2}\) conditions as well.

Two sentences with different truth\(_{w_1,w_2}\) conditions might have the same truth\(_{w,w}\) conditions. Indeed, if \( \alpha \) is a formula of the language of S5A\( F \) free of ‘\( \Box \)’ and ‘\( F \)’, and \( \alpha' \) the formula resulting from \( \alpha \) by the removal of all occurrences of ‘\( A \)’, then S5A\( F \) \( \vdash \) \( F A (\alpha \leftrightarrow \alpha') \), so that for any similarly related sentences \( \sigma \) and \( \sigma' \) of the interpreted object language for which we imagine the truth theory presented here to have been constructed that theory will yield as theorems:

\[
F A [\text{Tr}(\bar{\sigma}) \leftrightarrow \text{Tr}(\bar{\sigma}')] \\
\text{and} \\
F A [\text{Tr}(\bar{\sigma}) \leftrightarrow \sigma'] .
\]

Thus, for example, the condition for the truth (in the sense that matters) of ‘Grass is actually green’ is just that grass is green, and this condition is one
which might not have obtained, which fact can be expressed equivalently using either ‘\(\sim \square\)’ or ‘\(\sim A\)’. So there is a clear sense in which ‘Grass is actually green’ is contingent even though the \(\square\)-modalization of that sentence is true, namely the sense captured by ‘deeply contingent’. Similarly, the condition for the truth (in the sense that matters) of the sentences (S), (S'), and (S'') of Section 2 is just that if anyone uniquely invented the zip, the inventor of the zip invented the zip, and this is a condition which could not but obtain, which fact can be expressed using either ‘\(\square\)’ or ‘\(A\)’. Thus there is a clear sense in which all three sentences are necessary in spite of the falsity of the \(\square\)-modalizations of (S) and (S''), namely the sense captured by ‘deeply necessary’. In general, a true sentence \(\sigma\) is deeply contingent just in case its truth condition might not have obtained, that is – we suggest – just in case its truth condition does not hold of every world \(w\), that is, just in case \(\sim \sim A \sim\) is true. This discharges a promise made in Section 2.

THE NECESSARY ‘A POSTERIORI’ AND OTHER APPLICATIONS

In this section we suggest – tentatively – three further applications of the thought behind descriptive names: to the semantics of natural kind (and mass) terms, to the primary/secondary quality distinction, and to the meta-ethical theory sometimes known as subjective naturalism. This last application will highlight the distinction between assertive content and ingredient sense – the difference, that is, between that semantic feature in which sentences (S) and (S') of Section 2 agree and that in which they differ, as shown by their embeddings in \(\square\)-contexts.

Consider again the descriptive name ‘Julius’ of the man who actually invented the zip, and the proper name ‘Tom’ of that same man. Then it is an a posteriori truth that Julius is Tom and yet ‘Julius is Tom’ is necessarily true since ‘\(\square\) (Julius is Tom)’ is (modulo problems of contingent existence) true. So descriptive names give rise to necessary a posteriori truths. But clearly there is more to be said. Certainly ‘Julius is Tom’ is superficially necessary, and certainly that necessity is guaranteed by the fact that ‘Julius is Tom’ is an identity statement using names. But, equally clearly, ‘Julius is Tom’ is deeply contingent, and that contingency rests on the fact that while both names are in a sense rigid designators (since both retain their reference under changes in which world is considered as the ‘floating’ world), ‘Tom’
does, while ‘Julius’ does not, retain its reference under changes in which world is considered as the actual world.

These distinctions enable us to make sense of what might otherwise look like a disagreement over the modal status of ‘Julius is Tom’. For one party might say, ‘Julius, the man who actually invented the zip, could not have failed to be Tom even if he had failed to invent the zip’, while the other urged that in an imagined counterfactual situation in which Tom did not invent the zip, the speakers of our language would speak truly were they to say: Julius is not Tom. Apparent disagreement would be multiplied if some speakers came to know that Tom invented the zip and allowed this knowledge so to infect their use of ‘Julius’ that it too became a proper rather than a descriptive name of Tom. For although these speakers would agree with the rest as to the truth-values of all sentences free of ‘∃’, they would judge that in no sense is ‘Julius is Tom’ contingent; in the imagined counterfactual situation, ‘Julius’ would have a different meaning from that which it has in their language.

These apparent disagreements are reminiscent of more familiar apparent disagreements over the modal status of identities such as ‘Water is H₂O’, and we suggest that at least some light may be cast on the semantics of natural kind words by seeing them as analogous to descriptive names, rather than proper names, of chemical, physical, or biological kinds. Thus, suppose that ‘water’ is a descriptive name with its reference fixed by the description ‘the chemical kind to which that liquid belongs which falls from clouds, flows in rivers, is drinkable, colourless, odourless,...’ so that with w₁ as the actual world the reference of ‘water’ with respect to w₂ is that chemical kind of stuff which in w₁ falls from clouds, .... To understand ‘water’ it would not be necessary to know which chemical kind actually has those properties and so it would be an a posteriori discovery that water is H₂O; what is more, this would have to be recognized as a posteriori even by those who hold that all true identities using proper names express a priori truths. The true identity statement ‘Water is H₂O’ would be deeply but not superficially contingent, but for those among the chemically informed for whom ‘water’ had become another proper name of H₂O, ‘Water is H₂O’ would be contingent in no sense.

On the suggested view, one would hold that “water” was world-relative but constant in meaning’, and that in one sense, for worlds w₁ and w₂, ‘water is H₂O in w₁ and water is XYZ in w₂’, namely the sense which answers to the fact that ‘Water is H₂O’ is true w₁, w₁ and yet ‘Water is XYZ’ is true w₂, w₂. Yet
one would also hold that in a sense 'water is H₂O in all worlds (the stuff called 'water' in w₂ isn’t water)', namely that sense which answers to the fact that for w* as the actual world, ‘Water is H₂O’ is trueᵦᵦ for all worlds w, and ‘Water is XYZ’ is falseᵦᵦ for all words w. Thus on the suggested view, one would agree with Putnam (from whom these quotations have been taken) that ‘indexicality extends beyond the obviously indexical words and morphemes... words like “water” have an unnoticed indexical element’. But the suggested view is not quite Putnam’s view; he regards it as ‘plausible’ and ‘the correct route to take for an absolutely indexical word like “I”’, but incorrect for natural kind words and so, in particular, for ‘water’. In [21], he offers a reason for rejecting the suggested view, namely that if we imagine a possible situation in which the chemical kind which has the functional features H₂O has in the actual world is XYZ, and imagine further that in that world the word ‘water’ is replaced by ‘quaxel’, then the suggested view commits us to saying that two different words referring to two different liquids nevertheless have the same meaning and that this is ‘highly counterintuitive’. Putnam seems to be in error here: there is nothing more counterintuitive about the imagined case than there is about the following case. Suppose that ‘Julius’ is an abbreviation of ‘the actual inventor of the zip’ and imagine a counterfactual situation in which ‘Suiluj’ abbreviates that same description and in which someone other than Tom invented the zip; then ‘Julius’ and ‘Suiluj’ are different words referring to different men and yet they clearly have the same (indexical) meaning. For a more commonplace example, consider the words ‘I’ and ‘je’ as uttered by an Englishman and a Frenchman respectively: different words, different referents, but same meaning.

There is another possible objection to the suggested view, for it may be urged that attribution to speakers of competence with a word ‘water’ whose reference is fixed by the rather lengthy and vaguely specified description, ‘the chemical kind of stuff which...’ describes their use of that word as altogether too reflective. Might is not be that although these (loosely speaking) functional features of water operated as perceptual cues, the speakers were not very self-conscious and so did not recognize this fact? In response to this objection, we might modify the reference-fixing description by introducing a new word ‘waterish’ which is to be a (quasi-)functional word, rather than a natural kind word: one can imagine speakers being trained with this word in the presence of stuff which fell from clouds, and so on, and then
being introduced to the descriptive name 'water' via the reference-fixing description 'the chemical kind to which waterish stuff belongs'. Or equivalently, one might introduce a proper name 'water*' of a functional kind rather than a chemical kind, and introduce 'water' via the reference-fixing description 'the chemical kind which realizes water*'. It is worth noting that if 'water*' and 'water' were not given different phonological realizations in the language, then because in the usage of the chemically informed 'water' might have become a proper name of the chemical kind, the word 'water' (as we write the 'water'/'water*' neutral form) would be three-ways ambiguous in the population.

A final objection is more serious. Imagine that every speaker of the language containing 'Julius' had a visual confrontation with Tom and was told, 'This man is Julius'. Then one expects that 'Julius' would become a proper name of Tom. This is not, of course, to say that for the identity statement 'This man is Julius' or even for 'This man is called "Julius"' to be true, 'Julius' must be a proper name, but only that, given the knowledge which each speaker would now have (knowledge by acquaintance of Julius) it would be natural for the semantic function of 'Julius' to change. But now consider the fact that practically every speaker of our language has had a visual confrontation with (a sample of) the chemical kind H₂O and been told, 'This stuff (this chemical kind) is water (is called "water")'. Is it not unlikely that 'water' remains, in our language, a merely descriptive name of H₂O? A reply to this objection will depend upon claiming that physical ostension of a sample of H₂O accompanied by the words 'This stuff' ('the chemical kind here exemplified') is similar not to physical ostension of a man accompanied by the words 'this man', but rather to physical ostension of a screen accompanied by 'the man behind this screen', or even physical ostension of a zip accompanied by 'the inventor of this device'. Since we do not know whether such a claim can be defended, we are not confident that the suggested view is correct.

If the suggested view were adopted for 'water', then it would be worth considering an analogous treatment of the predicate '(is a) tiger'. Consider first that there is a predicate 'tiger*' whose correct application depends only on the gross physical characteristics of objects (in particular, in this case, on felinity, ferocity, size, stripedness, etc.) and then think of the predicate 'tiger' as being introduced in a way analogous to that in which descriptive names are introduced, namely by:
TWO NOTIONS OF NECESSITY

$x$ satisfies ‘tiger’ iff $x$ is of the biological kind whose members are actually stereotypically tiger*.

Or, in possible worlds terminology:

$x$ satisfies$_{w_1, w_2}$ ‘tiger’ iff $x$ is of the biological kind in $w_2$ whose members are stereotypically tiger* in $w_1$.

Given this view it will be knowable a priori that tigers are stereotypically tiger*, even though there is no contradiction involved in saying: tigers might all have been placid and stripeless.

The second application of the ideas of Sections 0–3 to a topic of philosophical interest is an application to the treatment of secondary quality words. It seems — oversimplifying a little here — that science has discovered that for a material object to be red is just for it to reflect light of a certain wavelength $\alpha$, and this makes good sense of our intuition that red things might have looked quite different to us, if, for example, our perceptual apparatus had been different. It also enables us to say, as we surely do not want any theory of the meaning of ‘red’ to prevent us from saying, that tomatoes, for example, would still have been red even if there had been no perceivers. Yet, it is clearly not the case that ‘red’ simply means ‘reflects light of wavelength $\alpha$’; indeed, some language users are inclined to say that had red objects — objects which reflect light of wavelength $\alpha$ — looked quite different and had objects reflecting light of wavelength $\beta$ looked to us the way tomatoes and pillar boxes actually look, then we would have used ‘red’ to describe objects which reflected wavelength $\beta$, and would have done so without changing the meaning of ‘red’. What descriptive names suggest is a way of holding to both of these prima facie conflicting intuitions: we suggest that ‘red’ is analogous to a descriptive name of a physical property (having to do with reflecting light of certain wavelengths) whose reference is fixed by a phenomenal description. For obvious reasons the description cannot speak of tomatoes and pillar boxes; rather we shall introduce a predicate ‘red*’ which — according to one’s preferences in the philosophy of perception — is either a predicate of physical objects (or their surfaces) whose correct application turns solely on the way the object looks, or else a predicate of sense data. Then ‘red’ is introduced via either:

$x$ satisfies ‘red’ iff $x$ has that physical (reflective) property which actually standardly results in objects being red*.

or:
$x$ satisfies 'red' iff $x$ has that physical (reflective) property which \textit{actually} standardly produces red* sense data in perceivers.

The upshot of either mode of introduction is that the sentence ‘$\forall x (x$ is red $\rightarrow x$ reflects light of wavelength $\alpha$)’ is true, as is the sentence ‘$\mathcal{F} A (\forall x)(x$ is red $\rightarrow x$ has whatever reflective property standardly results in objects being red*/objects inducing red* sense data)’, while neither of the sentences resulting from interchanging ‘$\square$’ and ‘$\mathcal{F} A$’ in these sentences is true. Again, it is no part of our position that the suggested view is the ultimately correct view about the way ‘red’ functions in English; it is part of our position that no dispute amongst theorists of perception over secondary quality words should proceed without taking account of the \textit{prima facie} coherence of the suggested view.\textsuperscript{36}

Our final application to an area outside philosophical logic or the philosophy of language of the ideas discussed in earlier sections of this paper is to the viability of subjective naturalism (henceforth: subjectivism) as a meta-ethical theory. Again, we are not trying to show that the theory is correct, but to show how it may be formulated so as to avoid certain familiar objections. We use ‘Dis’ as a second-level predicate reading ‘Dis($F$)’ as ‘$F$-ness evokes disapproval in me’ (so that sentences involving ‘Dis’ are indexical); we assume that the subjectivist has some purely psychological account of this predicate ‘Dis’ (not cast in terms of thinking wrong) and that his suggested analysis for judgments of the form ‘(particular) action $a$ is wrong’ runs: $(\exists F) (\text{Dis($F$) } \land F(a))$. Many familiar objections to such a position fall under the general rubric of subordinate context objections. Let us, to illustrate, take the objection in the form in which the context is conditional. It runs thus. The sentence ‘If I disapproved of that act of (say) rendering assistance – call it $a$ – then $a$ would be wrong’ does not express a necessary truth, yet on the theory in question it, or something very much like it, should express such a truth, inheriting its necessity from the necessity of ‘If $a$ were wrong, $a$ would be wrong’ by substitution of analysans for analysandum. The line of reply we wish to investigate may initially be explained as one denying that the substitutability of analysans for analysandum in arbitrary sentential contexts is required of a claim of the sort the subjectivist is making. To pursue this line of thought, one needs to distinguish between the \textit{ingredient sense} and the \textit{assertive content} of a sentence. Such a distinction, and the terminology we use for marking it, is suggested by a passage in Dummett [7]:
In this case, however, we must distinguish, as we have seen, between knowing the meaning of a statement in the sense of grasping the content of an assertion of it, and in the sense of knowing the contribution it makes to determining the content of a complex statement in which it is a constituent: let us refer to the former as simply knowing the content of the statement, and to the latter as knowing its ingredient sense.\(^{37}\)

The bearing of this distinction on the defense of subjectivism against subordinate context objections is that such objections assume the subjectivist to be aiming at renderings of ingredient sense; yet the philosophical interest of subjectivism would certainly survive a retreat to the weaker position in which it was assertive content only that was at issue, since if this weaker position were indeed correct, then everything that can be said with the aid of moral vocabulary could be said in non-moral vocabulary. The fact that the translation from moral language into language about the speaker’s feelings is not a uniform translation — translating a whole by concatenating the translation of the parts — is neither here nor there.

To flesh out the imagined position a little, something should be said about how our assertive content subjectivist proposes to deal with those complex constructions, like the conditional, that show ingredient sense subjectivism to be mistaken. The following principle looks plausible. The assertive content of ‘\(a\) is wrong’ (where the blanks indicate the surrounding sentential context) is given by ‘\((\exists F)(\text{Dis}(F) \land \text{F(a)})\)’; thus for ‘If \(a\) is wrong then its perpetrator should be punished’ not the implausible paraphrase ‘If \(a\) has features evoking disapproval in me, then its perpetrator should be punished’,\(^{38}\) but the rather better candidate, ‘Certain features of actions evoke disapproval in me and if \(a\) has (some of) those features, then its perpetrator should be punished’. This suggestion is much better because it represents the perpetrator’s deserts as dependent on the kind of act he has performed, rather than on the speaker’s feelings — though it is by reference to these feelings that the kind of act is specified. The difference between the two is a matter of what precisely is made conditional on what. Similar manoeuvring enables the subjectivist to escape a number of otherwise embarrassing objections. For instance, if he says (rather obscurely) that being wrong is just a matter of being disapproved of by him, then it may look as though he is committed to saying that nothing will be wrong after his death: for certainly nothing will then be disapproved of by him. However, things will continue to have qualities or characteristics, and some of these may well be such that they now evoke disapproval in him. The important thing about the present strategy of paraphrase — which delivers not the unwanted ‘It will be the case
that \((\exists F)(\text{Dis}(F) \land F(a))\) but instead \(\left((\exists F)(\text{Dis}(F) \land \text{it will be the case that } F(a))\right)\) for 'a will be wrong' — is that it keeps the futurity where it belongs: on the object’s possession of certain characteristics, and not on the subject’s having of certain feelings. \(^{39}\)

To see the relation between the meta-ethical theory just sketched and the other issues of this paper, observe that instead of retreating from a claim to provide ingredient sense to the position in which only a rendering of assertive content is claimed, the stronger claim might be sustainable if the subjectivist’s paraphrase is to be given in an ‘actually’-enriched language, since then he may offer for 'a is wrong', \(\left((\exists F)(A(\text{Dis}(F)) \land F(a))\right)\), dealing with subordinate context objections — at least of the intensional conditional and modal variety \(^{40}\)— by exploiting the logic of ‘A’, which protects what is in its scope from the pernicious world-shifting effect of modal operators in whose scope it lies, instead of giving a special wide-scope rendering afresh every time, as suggested in the previous paragraph, to the \(\left((\exists F)(\text{Dis}(F)) \land \right)\) part of the paraphrase. In more familiar terms, the present suggestion is that we see the speaker’s feelings as entering into a reference-fixing, rather than a sense-giving, role for the predicate 'wrong'. The situation is not at all dissimilar to the secondary quality case already discussed. There, it was suggested that those physical properties which underlie the colour appearances of objects in the actual world are what is relevant to the applicability of a colour predicate even with respect to counterfactual situations in which those properties underlie quite different — or nonexistent — colour appearances; here, the suggestion is that it is the non-moral qualities on the basis of which disapproval is actually felt which are the properties relevant to the applicability of moral predicates with respect to counterfactual situations in which other (or no) feelings may be aroused by those properties. In fact, some objections to carelessly formulated subjectivism are strikingly similar to objections raised against a carelessly formulated doctrine of secondary qualities; we have already mentioned that no account of secondary qualities can be correct if it denies that objects would still be coloured in the absence of perceivers — and we showed above how to reconcile this with the thought that the visual experiences of actual perceivers remain an ineliminable part of (= must figure in a specification of) the meaning of terms like 'red'. The meta-ethical analogue to the objection thus sidestepped in Moore’s objection to subjectivism running: 'a is wrong' cannot mean 'I disapprove of a' (or 'a has characteristics which evoke disapproval in me') because the latter entails 'I
exist' while the former does not. This objection does not touch the present formulation of subjectivism, for the entailment does not hold when an ‘actually’ is inserted before ‘evoke’ in the parenthesized version of the paraphrase: just as perceiverless worlds contain coloured objects because they contain objects with properties which actually cause colour experiences, so I-less worlds may be full of wrongdoings because they contain actions with properties which in the actual world evoke my disapproval. 41

Indeed, not only has our excursion into ethical theory not removed us too greatly from the other applications presented, we are not even very far from the ever-recurring story of the invention of the zip. For in Section III of RC, Evans makes use of Dummett’s distinction between (assertive) content and ingredient sense – which he refers to as the content of an assertion of a sentence and the proposition expressed by that sentence, respectively – in the course of his discussion of descriptive names and the contingent a priori. Evans observes that sentences of the forms:

(S1) Julius is F
(S2) The inventor of the zip is F.

have the same assertive content but differ as to ingredient sense. The content of the two is the same because what is required for the TRUTH (in the sense of Section 3) of an utterance of either is just what is required for the TRUTH of an utterance of the other: thus what a speaker commits himself to by an outright assertion of (S1) is precisely the same as what he commits himself to by an outright assertion of (S2). As we have already seen, however, sentences like (S1) and (S2) give vastly different results when embedded in □-contexts, which justifies Evans’ claim of difference in ingredient sense. (These points can all be seen very clearly if we take, instead of (S1), the result of inserting ‘actual’ before ‘inventor’ in (S2), and compare this with (S2); this new sentence and (S1) do not differ with respect to any of the semantic properties we have mentioned in the present section. 42)

We close this section by noting that, just as it is possible to characterize the notions of deep and superficial necessity and contingency in terms of the truth of sentences of the language of S5A.F (Section 2, above), so a similar characterization of the relations of sharing assertive content and sharing ingredient sense falls out if Sections 1 and 3 are shaken a little. Since ‘F □’ (or equivalent) is the weakest prefix for which a substitutivity property like (SE) is forthcoming, for sentences σ and σ’ to be alike in ingredient sense is
just for $\square (\sigma \leftrightarrow \sigma')$ to be true. On the other hand, for $\sigma$ and $\sigma'$ to have the same assertive content is for $\square \sigma \leftrightarrow \square \sigma'$ to be true, for any $w$, which is guaranteed by (and guarantees) the truth of $\square A(\sigma \leftrightarrow \sigma')$.\footnote{43}

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\section*{Notes}

\footnote{*}{We are grateful to Gareth Evans, Frank Jackson, David Lewis and Christopher Peacocke for comments on earlier versions of this paper.}

\footnote{1}{For some further comments on these two notions of validity see LA.}

\footnote{2}{Or, more correctly: all instances of the following forms. For brevity, we continue to speak of axiom-schemata simply as axioms. In LA particular axioms were used, with a rule of uniform substitution. However, this is not convenient in the present context as uniform substitution does not preserve validity in the logic we present in the following sections as an extension of SSA. We have also omitted a redundant axiom listed in LA.}

\footnote{3}{That is, $(W, x, V) \sim (W', x', V')$ iff $W = W'$ and $V = V'$.}

\footnote{4}{See Aqvist \cite{1}, Segerberg \cite{24}, van Fraassen \cite{27}. The equivalence between the two ways of presenting the model theory is remarked on by Kamp in \cite{12} in connexion with tense logic where 'Now' plays the role of 'Actually'. Kamp operates with the different conception of validity mentioned in Section 0, and does not consider the temporal analogue of 'Fixedly'.}

\footnote{5}{In the notation of two-dimensional modal logic the operators $\mathbf{1}-\mathbf{4}$ can be defined neatly:}

\footnote{6}{The expression 'two-dimensional' nevertheless suggests a helpful way of thinking about some of the issues of the present paper. Suppose, for simplicity, that our model $W$ has just three worlds of which $w^*$ is designated. Then we can assign to a sentence a three-by-three array of 'T's and 'F's in which the top row corresponds to the model $W$ and the other two rows correspond to the variants of $W$. For some uses of this visual aid see below at Notes 8 and 15, and cf. Stalnaker \cite{25}.}

\footnote{7}{Symmetric as well, even.}

\footnote{8}{For a discussion of the provability of instances of (T4) see \cite{11}, Section 1.}

\footnote{9}{In terms of the matrices introduced at Note 5 the fact that a matrix has 'T' across the top row and down the left hand column does not ensure that it has 'T' everywhere.}

\footnote{10}{The symbols '$\sigma$' and '$\tau$' are to range over the sentences of an interpreted language and so are to be confused neither with the propositional variables of the formal language in which the logic SSA is cast nor with metalinguistic variables (for which we have been using '$\alpha$', '$\beta$', etc) over arbitrary formulae of that language. We follow the common procedure of letting autonomy resolve matters of use-mention in discussions of the formal language while using corner-quotes to that end in the discussion of an interpreted language.}

\begin{align*}
\models^x_\gamma & \mathbf{1} \alpha \text{ iff for any } z \in W, \models^z_\gamma \alpha \\
\models^x_\gamma & \mathbf{2} \alpha \text{ iff for any } z \in W, \models^z_\gamma \alpha \\
\models^x_\gamma & \mathbf{3} \alpha \text{ iff for any } z \in W, \models^z_\gamma \alpha \\
\models^x_\gamma & \mathbf{4} \alpha \text{ iff for any } w, z \in W, \models^{wz}_\gamma \alpha.
\end{align*}
The idea of reference-fixing (as opposed to sense-giving) descriptions is from Kripke [14].

These two features of proper names are emphasized particularly in Peacocke [18] and McDowell [16], respectively. The view of proper names mentioned is plausible but not unassailable; one might want to acknowledge that ordinary proper names of spatio-temporally remote objects are more like descriptive names. And the notion of de re propositional attitudes may itself come under attack.

Because we have here an instance of a theorem of S5\(\mathcal{A}\).\

The observation recorded in this sentence was made to us some years ago by David Lewis, in the course of an exposition and development of some then unpublished ideas of Stalnaker’s (now published in [25]).

A sentence which is actually true, and so has ‘T’ in the top left corner of its intended matrix, is a superficially contingent truth if there is nevertheless an ‘F’ somewhere along the top row. It is deeply contingent if there is an ‘F’ somewhere along the diagonal from top left to bottom right. It is bi-contingent if there is merely an ‘F’ somewhere in the matrix.

At p. 320 of [25] Stalnaker offers the complex operator ‘\(\Box\; \dagger\)’ as an a priori truth operator, where, in two dimensional notation,

\[
\models^x \dagger \alpha \iff \models^y \alpha
\]

Thus

\[
\models^y \Box \dagger \alpha
\]

iff

\[
\forall y' \models^y \dagger \alpha
\]

iff

\[
\forall y' \models^y \dagger \alpha
\]

iff

\[
\models^y \mathcal{F} A\alpha.
\]

In terms of the model-variance semantics of Section 0, this suggestion amounts to distinguishing a designated model just as in a single model we have a designated world: the designated model is the one with the ‘right’ world as its designated world, and ‘R’ keeps us looking at the same world but shifted over so as to be viewed from the perspective of the designated model.

For some remarks on all this, see [10], Section 3.

Thus we must dissent from Evans’ claim that ‘the contingency of (S) crucially depends upon the fact that “Julius” is a referring expression’. (RC, p. 175).

This first way was suggested by Gareth Evans.


See [9].

The appearance, in these axioms, of quantifiers (over sentences) should not be greeted with alarm. We assume only that one may infer any instance of a universally quantified formula from that formula, and that, since sentences can fail neither to exist nor to be sentences, the quantifiers in question commute with the operators ‘\(\mathcal{F}\)’ and ‘\(\Box\)’.

We refer to the objection (due to Wallace) that the modalized truth theory is ridden with falsehoods since any linguistic expression has the semantic properties it has only contingently; the reply we have in mind is most fully worked out in Peacocke [19] (Section II). It consists in thinking of the language \(L\), as it is referred to in such metalinguistic predicates as ‘true-in-L’, as being identified by its semantic features, it being then a contingent question which (if any) population actually has \(L\) as its language. This line of reply was anticipated by Baldwin, in [2], pp. 84f.

The quotation is from Peacocke [19], p. 490; the word ‘predicates’ should presumably read ‘predications’.
Actually, it suffices to add only one (either one) of the two conditionals conjoined
into (T4) as a new axiom.

Thus, on one natural interpretation at least, Plantinga is in error when he writes (at
p. 46 of [20]), 'A proposition is true in the actual world if it is true; it is true in w if it
would have been true had w been actual.'

This provides the illumination, promised at the end of Section 0, of the relation
between truth in a possible world and truth with respect to a possible world.

See Kripke [14] for the notion of 'rigid designator'.

See [22], p. 231; the remaining quotations are from pp. 234 and 245.

At p. 704 of [21], Putnam prefers an example using 'elm' and 'beech'. If 'elm' really
is semantically akin to 'water' then the 'elm'/beech' example cannot show anything
which the 'water'/'quaxol' example doesn't show; if 'elm' is not semantically akin to
'water' then the most that the 'elm'/beech' example could show would be that the
suggested view would not be correct for 'elm', whatever its merits for the case of 'water'.

CL McDowell [16], p. 177.

We do not have a fully worked out theory of when knowledge is knowledge by
acquaintance; it is enough for present purposes that such knowledge be knowledge de
re concerning Julius; see again the account of e-salience in Section 2.

The points of the present section made about all predicates mentioned can be made
either as taking words like 'tiger' not as predicates but as (descriptive) names of natural
kinds (or properties), or else as predicates in the strict sense, whose satisfaction-condi-
tions are specified in a way analogous to that in which the denotation of a descriptive
name is specified.

Not because such things might be other than red - this is taken care of by the
presence of 'actually' - but because users of 'red' need have had no experience of these
particular things.

Re-reading Kripke [14] we find at footnote 71 and the corresponding text on
p. 331 a very clear anticipation of the present suggestion for secondary quality words.

Christopher Peacocke has pointed out to us that a slightly different 'actually'
operator is required if we wish to use a 'descriptive names' strategy to meet an objection,
mentioned for example in [4], to the use of equivalence relations 'is same coloured
with' or 'is same hairstyled with' rather than an ontology of colours or hairstyles. For
example if 'F' is a predicate applicable on the basis of hairstyle, and if Whitlam has his
hair thus styled, then to avoid an ontology of styles we might say

\[ x \text{ satisfies } 'F' \text{ iff } x \text{ is same hairstyled with Whitlam.} \]

The objection in this case is that it is neither necessary nor sufficient for the truth of
'Fraser might have been F' that Fraser and Whitlam might have had the same hairstyle.
What is needed to meet this objection is an 'actually' operator which marks a particular
argument place:

\[ x \text{ satisfies } 'F' \text{ iff } A_y^{\text{ Whiltam}} (x \text{ is same hairstyled with } y). \]

In possible worlds terminology:

\[ x \text{ satisfies } 'F' \text{ with respect to world } w \text{ iff } x \text{ in } w \text{ is same hairstyled with } \]

\[ \text{Whiltam in the actual world.} \]

[7], pp. 446f.

We ignore the fact that eventually the 'should' in the consequent will require
paraphrase.

The past-tense analogue of the objection to which this is a response may be extracted
from Moore [17], Chapter 3. The suggestion offered here for what the subjectivist
should say about 'a will be wrong' is what is normally said in reply to Moore (for
example, Stevenson in [26]), though it is not generally recognized that the question
of why substitution of 'a is disapproved of by me' for the allegedly synonymous 'a is wrong' does not yield synonymous results in the context 'It will be the case that ——.' — that this question needs to be explicitly addressed.

Tense is a separate, though precisely analogous, problem; to deal with cases like 'a will be wrong' in the present style we should need to add a 'now' operator and use 'actually now' as a prefix to 'Dis'. Indirect speech and propositional attitude contexts present yet other difficulties; some help may come from thinking of 'Dis' as meaning not 'evoking disapproval in me' but 'evoking disapproval in me*', where 'me*' is the appropriate form of 'I*' a floating first person pronoun (somewhat along the lines of Castañeda's 'He*' in [5]). The sketch of assertive content and ingredient sense subjectivism in the text is as brief as is possible for purposes of illustration. For example, there are in fact reasons — which we do not go into — for the existential quantifier over properties used by both theories to be interpreted as a uniqueness quantifier (thus we should think of 'Dis' as applying to a maximal disapproval-evoking property).

The 'descriptive names' strategy could, of course, be employed independently of subjectivism, and it has some appeal. But we do not underestimate the difficulties which it faces when one tries to give non-moral 'reference-fixing descriptions' for particular virtues such as justice.

Note that the obvious truth of such sentences as 'The actual inventor of the zip might not have invented the zip' shows that Kripke's modal and counterfactual subordinate context objections to sense theories of proper names (in [14]) fail to show that the sense of a proper name isn't captured by an 'actually'-containing description. Fortunately, as our brief discussion of e-salience in Section 2 suggests, there are other ways of distinguishing proper from descriptive names.

Some might wish to complicate this simple criterion in order to accommodate the view that, for co-denoting proper names 'a' and 'b', it is possible for 'Fa' and 'Fb' to differ in content in spite of the truth of $\lnot F(A \leftrightarrow B)$ (and $\lnot F(\Box A \leftrightarrow B)$, even). We should also mention that the present account of the sense/content distinction is only meant to elucidate the distinction as it arises in connexion with the contexts expressible in the language of $S\Delta A \exists \forall$. For an example of a different sort consider Evans' discussion in RC of what are sometimes called the presuppositions of sentences containing definite descriptions. An even more straightforward example would be Quine's account of 'any' (in [23], Section 29) as a universal quantifier having maximal scope; such an account precludes the regimentation of a conditional with 'any' in its antecedent as the conditional construction of the regimentation of the antecedent with that of the consequent. It is therefore not an account of ingredient sense.

BIBLIOGRAPHY