The two-dimensional semantic framework, with its two-dimensional matrices of truth values, was developed for tense logic by Frank Vlach (1973), building on work by Hans Kamp (1971), and for modal logic by Lennart Åqvist (1973), Krister Segerberg (1973), and Bas van Fraassen (1977). Other antecedents of the contemporary use of the framework are found in formal work on context-dependence by Richard Montague (1968) and David Lewis (1970) and especially in David Kaplan’s distinction between character and content in ‘Demonstratives’ (published in 1989, but circulating in manuscript from 1979). The character of a sentence is a function from contexts to contents, or equivalently, a function from contexts and possible worlds to truth values. It could be represented by a two-dimensional matrix with contexts along the vertical dimension and possible worlds along the horizontal dimension.

Robert Stalnaker’s paper ‘Assertion’ (1978) was conceived as a contribution to pragmatics rather than to formal semantics – an investigation of context-dependence and informativeness for utterances rather than of modal logic. He was concerned with the way that context helps to determine what is said in an utterance (its content) and with the way that speech acts alter context. Stalnaker took over the idea of a two-dimensional matrix from the semantics of two-dimensional modal logic; but he used the formalism in a very different way. Indeed, Stalnaker’s use of the two-dimensional framework is different both from its use in modal semantics and from the use suggested by Kaplan’s distinction between character and content. In Stalnaker’s use, the rows of the matrix represent the content that an utterance would have, not only in different contexts, but also with different meanings assigned to the expressions used. Sometimes, Stalnaker proposes, what is communicated by an utterance is best represented by the function from worlds to truth values that corresponds to the leading diagonal of such a
matrix (1978, pp. 91–92). In order to see how an utterance of ‘That is Elizabeth Anscombe’ or ‘Hesperus is identical with Phosphorus’ can be informative despite having a content that is a necessary truth, we need to consider the content that the utterance would have under different contextual or even semantic assignments of reference (to the demonstrative ‘that’ or to a name).

In ‘Two Notions of Necessity’ (1980), Martin Davies and Lloyd Humberstone were concerned with neither context-dependence nor pragmatics. Building on Humberstone’s earlier work with John Crossley on modal logic with an ‘Actually’-operator (Crossley and Humberstone, 1977) and also on Gareth Evans’s paper, ‘Reference and Contingency’ (1979), they proposed that, in a two-dimensional matrix, truth on the horizontal and truth on the diagonal correspond to two notions of necessity that Evans called ‘superficial’ (the necessity expressed by ‘□’) and ‘deep’ respectively. They were thus able to give formal expression to Evans’s claim that apparent examples of contingent a priori truths are superficially contingent but deeply necessary.

In ‘Assertion’, Stalnaker briefly suggested that the modal combination ‘□†’, expressing truth on the diagonal (and so equivalent to Davies and Humberstone’s combination ‘\(\mathcal{F}A\)’ expressing Evans’s deep necessity), ‘can be understood as the a priori truth operator’ (1978, p. 83). But Stalnaker subsequently rejected that suggestion (1999, pp. 15–6, 2003, p. 199) and it clearly does not sit very happily with a use of the two-dimensional framework that allows variations in meaning. It is plausibly a priori that all triangles have exactly three sides. But, once we allow the vertical dimension to take into account the fact that ‘triangle’ might have meant any closed figure with at least three sides, we also allow that ‘All triangles have exactly three sides’ will not come out true everywhere along the diagonal.

On Stalnaker’s view of reference, an ordinary proper name could not have had a different reference without having a different meaning. Thus, so long as semantic properties are held constant, an a posteriori true identity statement using ordinary proper names, such as ‘Cicero is Tully’, will be true throughout the two-dimensional matrix. But because Stalnaker allows for differences in meaning along the vertical dimension, he is able to register the
informativeness of such a statement by allowing that it is not true everywhere on the diagonal. Indeed, even an utterance of the \textit{a priori} true sentence ‘All triangles have exactly three sides’ could be informative to someone who thought that ‘triangle’ meant any closed figure with at least three sides; and Stalnaker is able to register this informativeness in the same way.

Davies and Humberstone agree with Stalnaker that, so long as meaning is held constant, ‘Cicero is Tully’ is true throughout the two-dimensional matrix. But, unlike Stalnaker, they do not attempt to register either its informativeness or its \textit{a posteriority} within the formal apparatus of the two-dimensional framework. Davies and Humberstone report that they have not noticed any examples of truths in their modal language that are \textit{a priori} but not deeply necessary in Evans’s terminology, that is, are not true on the diagonal. But, they allow, sentences that are true on the diagonal include some \textit{a posteriori} truths as well as \textit{a priori} truths (1980, p. 10). Thus, despite major differences in the uses to which they put the two-dimensional framework, Stalnaker and Davies and Humberstone agree in not taking truth on the diagonal to coincide with \textit{a priori}.

But more recent advocates of the framework, Frank Jackson and David Chalmers, do press for that identification.

In a wide-ranging critical notice of Susan Hurley’s book, \textit{Natural Reasons}, Jackson (1992) explains his view that conceptually necessary truths, such as ‘Water = water’, and \textit{a posteriori} necessary truths, such as ‘Water = H2O’, do not ‘differ in the kind of necessity they possess’ (1992, p. 482). The difference is, rather, in how we discover that these are necessary truths. Our concept of water does not tell us whether it is true that water is H2O. For, according to Jackson, our concept of water is ‘the concept of that which actually falls from the sky, is essential to life, is odourless and colourless, and so on’ (1992, pp. 483–484). So, in order to know whether water is H2O, we need to know whether H2O, or XYZ, or some other stuff, falls from the sky, is essential to life, and so on. Clearly that is an \textit{a posteriori} matter. But, if ‘Water = H2O’ is true then it is necessarily true; and the same goes for ‘Water = XYZ’. For ‘water’ is a rigid designator: ‘Water in any given possible world is the stuff which does the falling and so on in the actual world regardless of whether it does the falling and so on in the given world’ (1992, p. 484).
Jackson’s account of the modal and epistemic properties of
‘Water is H₂O’ is very close to a suggestion made by Davies and
Humberstone (1980, pp. 18–19):

[S]uppose that ‘water’ is a descriptive name with its reference fixed by the descrip-
tion ‘the chemical kind to which that liquid belongs which falls from clouds, flows
in rivers, is drinkable, colourless, odourless, . . .’ so that with w₁ as the actual
world the reference of ‘water’ with respect to w₂ is that chemical kind of stuff
which in w₁ falls from clouds . . . To understand ‘water’ it would not be necessary
to know which chemical kind actually has those properties and so it would be an
a posteriori discovery that water is H₂O . . . The true identity statement ‘Water
is H₂O’ would be deeply but not superficially contingent [that is, it would not be
true on the diagonal but it would be necessary in the sense expressed by ‘□’].

But there is an apparent difference between Jackson and Davies and
Humberstone over the question of how many notions of necessity
are in play. Jackson is concerned that there should be a single notion
of necessity, and especially that there should not be a distinction
between conceptually possible worlds and metaphysically possible
worlds. Davies and Humberstone agree that there is a single set of
possible worlds, but they have two notions of necessity. If ‘water’
is equivalent to an ‘actually’-embedding description, as Jackson
supposes, then ‘Water is H₂O’ is superficially necessary (true on the
horizontal) but not deeply necessary (not true on the diagonal). But
‘Water is water’ is both superficially and deeply necessary. Jackson
focuses on the single notion of necessity that the two sentences share
– the necessity expressed by ‘□’, truth on the horizontal. The fact
that ‘Water is H₂O’ is not true on the diagonal does not correspond
to its failure to participate in a second kind of necessity, but to its
failure to be a priori true (see further Jackson, 1994, 1998).

In The Conscious Mind: In Search of a Fundamental Theory
(1996, pp. 56–65), Chalmers gives an account which, while similar
to Jackson’s, also seems to differ in important respects. An expres-
sion, such as ‘water’, is associated with two functions from possible
worlds to referents. One function, the primary intension of the
expression, specifies the referent of the expression if the world turns
out to be this way or that – if this or that possible world turns out to
be the actual world. The value of this function for any ‘actual-world
candidate’ is, Chalmers says, an a priori matter even though it is an
a posteriori matter which possible world is actual (1996, p. 58). The
other function, the secondary intension, specifies the referent of the
expression in other possible worlds given its referent in the actual world. Thus, it turns out *a posteriori* that the referent of water in the actual world is H₂O and, because ‘water’ is a rigid designator, this function assigns H₂O as the referent of water in all other possible worlds.

The primary intension of a sentence corresponds to the diagonal in a two-dimensional matrix, while the secondary intension corresponds to the top row of the matrix, the row with our world as the actual-world candidate. The upshot, which illustrates the general pattern, is (roughly speaking) this. The sentence ‘Water is the dominant clear, drinkable liquid in the environment’ is *a priori* true and true on the diagonal (but not on the horizontal), while ‘Water is H₂O’ is *a posteriori* true and true on the horizontal (but not on the diagonal). Chalmers says that we can ‘think of the primary and secondary intensions as the *a priori* and *a posteriori* aspects of meaning, respectively’ (1996, p. 62). And with these two aspects of meaning go two notions of conceptual truth, or truth in virtue of meaning, and two notions of necessary truth.

There are three apparent differences between this account and Jackson’s. First, Jackson explicitly adopts a description theory of reference. But Chalmers says that the coincidence of *a priori* with truth on the diagonal does not depend on any such commitment (pp. 58–59). The primary intension of ‘water’ works the way it does, not because the reference of ‘water’ is fixed by a description like ‘the dominant clear, drinkable liquid in the environment’, but because (pp. 59–60): ‘Given that we have the ability to know what our concepts refer to when we know how the actual world turns out, then we have the ability to know what our concepts would refer to *if* the actual world turned out in various ways.’

Second, Jackson says that the difference between conceptual necessity and *a posteriori* necessity is not a difference between two kinds of necessity and does not reflect a difference between two kinds of possibility for worlds. But, while Chalmers agrees that there is ‘only one relevant kind of possibility of worlds’, he does allow that there are ‘two varieties of possibility and necessity [that] apply . . . to statements’ (1996, p. 63).

Third, Jackson does not build anything epistemic into the possible worlds that are ranged along the vertical and horizontal
dimensions of two-dimensional matrices. The connection between truth on the diagonal and a priori flows from Jackson’s views about understanding together with his commitment to a description theory of reference. But Chalmers’s use of the expression ‘ways the actual world might turn out’ (ibid., p. 63; emphasis added) at least suggests a more epistemological conception of primary intensions, and this is developed explicitly in subsequent papers. Thus, for example, in ‘On Sense and Intension’ (2002), primary intensions become epistemic intensions, while secondary intensions become subjunctive intensions. An epistemic intension is defined over epistemic possibilities or ways the world might be for all we know a priori, while a subjunctive intension is defined over counterfactual metaphysical possibilities.

If epistemic intensions are thought of primitively as ways the world might be for all we know a priori, then it is immediate that a sentence has an everywhere-true epistemic intension if and only if it can be known to be true a priori. But we cannot regard an epistemic intension, so conceived, as corresponding to the diagonal in a two-dimensional matrix. We can, of course, range epistemic possibilities along the vertical dimension and metaphysical possibilities along the horizontal dimension. But since epistemic possibilities are understood as different from metaphysical possibilities, no clear sense can be given to the idea of the ‘diagonal’ in such a matrix. The alternative is to start out by regarding an epistemic intension as corresponding to the diagonal in a two-dimensional matrix with metaphysical possibilities ranged along both the vertical and the horizontal dimension. But then substantive philosophical work is required in order to show that epistemic intensions have the desired connection with a priori. To the extent that this more familiar alternative approach is adopted, the third apparent difference between Chalmers’s position and Jackson’s is reduced, though other differences remain.

As this brief survey reveals, the two-dimensional semantic framework has been interpreted and used in a striking variety of ways. In the early work of Vlach on tense logic, the framework was, of course, given a temporal interpretation. In work on modal logic, and in the use of the framework by Davies and Humberstone, it receives a simple modal interpretation – possible worlds are ranged...
along both dimensions – and no attempt is made to encompass context-dependence. If Kaplan’s notions of character and content are set in the two-dimensional framework then, while we still have possible worlds along the horizontal dimension, it is contexts rather than worlds that are ranged along the vertical dimension. We might call this a contextual interpretation of the framework. In the terminology of Humberstone (this volume), we move from homogeneous cases to a heterogeneous case of two-dimensionality in which the notion of a diagonal has no application. We can take a step back in the direction of homogeneity by including in the representation of contexts an index for a possible world – the actual world, perhaps conceived as the world of the utterance by analogy with the speaker or the time of the utterance. But taking this step is problematic in a number of respects. First, unless we commit ourselves to counterpart theory there is no unique possible world in which an utterance takes place. Second, it is far from obvious that which world is actual impacts on the intuitive notion of ‘what is said’ in the same way that who the speaker is, or what the time is, does. Third, introducing possible worlds as an aspect of contexts of utterance runs the risk of assigning some kind of necessity to statements such as ‘Someone is speaking’.

Stalnaker focuses on utterances in contexts, but he extends the notion of contextual variation to include, not only differences in the speaker, the time, the place, or the objects being demonstrated, but also differences in the semantic properties of the expressions used. He calls this a metasemantic interpretation of the framework; but there is more than one way to draw the metasemantic versus semantic contrast. In a sense, any use of the framework in which the meanings of expressions are held constant, rather than being varied, could be called semantic rather than metasemantic. So Davies and Humberstone could be described as adopting, not only a simple modal interpretation, but also a semantic interpretation, of the framework. But Stalnaker (2003, pp. 192–195) has in mind a contrast between his use of the framework and the use of those who build variable reference into the semantics of names (and natural kind terms) by making a semantic connection between those expressions and definite descriptions. So it is Jackson, an advocate
of a description theory of reference, who most clearly stands on the opposite side of Stalnaker’s contrast from Stalnaker himself.

Jackson occasionally invokes the two-dimensional framework in discussion of examples of context-dependence (e.g. 1994, pp. 170–171). But for the most part, his use of the framework is in line with a simple modal interpretation. The same possible worlds are ranged along both dimensions and these worlds are conceived as metaphysically possible rather than as epistemically or conceptually possible. In contrast, as we have already noted, Chalmers uses the framework to capture epistemic and metaphysical (a priori and a posteriori) notions or aspects of meaning and of necessity. And he countenances an interpretation of the vertical dimension as primitively epistemic. An epistemic conception of the possibilities ranged along the vertical dimension also allows a distinctive treatment of context-dependence. If an epistemic possibility is a way that the world might be for all we know a priori, then an epistemic possibility needs to contain some locating or centring information. After all, for all I know a priori, the year might be 1990 or 2010, the place might be Chicago or Auckland, and I might be David Chalmers or Bob Stalnaker. This centring information can then be used in the account of the epistemic intensions of context-dependent expressions. This approach promises, at least for the most part, to avoid the threat that statements like ‘Someone is speaking’ will be assigned some kind of necessity (see Chalmers, 2002, pp. 154–157).

The two-dimensional framework has been interpreted in many different ways – temporal, modal, and contextual; semantic, meta-semantic, and epistemic. It has also been used for many different purposes – to provide models for modal logic, to regiment context-dependence, and to account for informativeness; to distinguish notions of possibility and to argue that we only need one notion. Those who use the framework incorporate many other theoretical commitments – maintenance of a gulf between modal and epistemological notions or a drive for convergence; direct reference or the description theory. And they have narrower or broader ambitions – ignoring or encompassing context-dependence; focusing only on linguistic meaning or extending to the content of thought.
Stalnaker, Jackson, Chalmers, Davies and Humberstone were the five main speakers at a workshop on the two-dimensional framework held at the Australian National University in Canberra over the weekend of 22–24 February 2002. Philip Pettit, Fred Kroon, Laura Schroeter, David Braddon-Mitchell, and Michaelis Michael were their respective commentators. With two exceptions, the papers in this special issue correspond to the talks and commentaries presented at the workshop. Stalnaker’s talk was based on his paper ‘Conceptual Truth and Metaphysical Necessity’ which has subsequently appeared in his collection *Ways a World Might Be* (2003). He has written a new paper for this special issue. Pettit’s paper addresses quite general features of Stalnaker’s position, but its origin as a commentary on ‘Conceptual Truth and Metaphysical Necessity’ shows through to some extent.

In July 1979, Gareth Evans wrote to Martin Davies commenting on a draft version of ‘Two Notions of Necessity’, and raising a number of objections to Davies and Humberstone’s use of the two-dimensional framework. Evans’s letter is published for the first time in this special issue, with the kind permission of Antonia Phillips.

**REFERENCES**


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